

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B:
1. All question are compulsory.
 2. Figures to the right indicate full marks.
 3. Students answering in the regional language should refer in case of doubt to the main text of the paper in English.

Q.1 Attempt **any three** of the following: 15

- a) Find Taylor's polynomial of order 0,1,2,3 generated by $f(x) = \sin x$ at $a = \pi/4$
- b) Define Maclaurin's series. Hence find the same for $\cos x$.
- c) Define mathematical model. Formulate the mathematical model for :
A motorboat goes upstream on a river and covers the distance between two towns on the riverbank in 12 hours. It covers this distance downstream in 10 hours. If the speed of the stream is 3 km/hr, find the speed of the boat in still water.
- d) Find the absolute, relative, and percentage errors if x is rounded- off to three decimal digits. Given $x = 0.005998$
- e) Define significant figures and significant digits. Round-off the numbers 865250 and 37.46235 to four significant figures and compute e_a , e_r , e_p in each case.
- f) Given a value of $\bar{x} = 2.5$ with an error of $\Delta\bar{x} = 0.01$, estimate the resulting error in the function, $f(x) = x^4$

Q.2 Attempt **any three** of the following: 15

- a) Perform 3 iterations of secant method to find root of
 $f(x) = \cos x - xe^x = 0$ in (0,1)
- b) Perform four iterations of Regula Falsi method for $f(x) = x^3 - 5x + 1$ such that the root lies in the interval (0,1).
- c) Write a note on finite difference operators and their properties.
- d) Construct the forward difference table and find value of function when $x = 5$, Given $x_0 = 0, h = 1, x_5 = 4, y_0 = 1, y_1 = 5, y_2 = 12, y_3 = 31, y_4 = 31, y_5 = 68$
- e) Find the smallest positive root of $f(x) = x^3 - 5x + 1 = 0$ by performing three iterations of Bisection Method.
- f) Find the unique polynomial $P(x)$ such that $P(2) = 1.5, P(5) = 4$ using Lagrange interpolation.

Q.3 Attempt **any three** of the following: 15

- a) Solve using Gauss Jordan method:
 $2x + y + z = 10 ; 3x + 2y + 3z = 18 ; x + 4y + 9z = 16$
- b) Find the value of y when $x = 0.1$, given that $y(0) = 1$ and $y' = x^2 + y$ by using Euler's Modified Method.

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Q.3

- c) Solve the equation: $x + 4y - z = 6$, $6x + y + z = 20$, $x - y + 5z = 7$ by using Gauss-Seidel Method
- d) For the following data determine $f'(2)$ and $f''(2)$
- | | | | | | | |
|---|----|----|----|----|----|----|
| X | 10 | 11 | 12 | 13 | 14 | 15 |
| Y | 8 | 4 | 7 | 6 | 15 | 2 |
- e) Evaluate using Trapezoidal rule : $\int_1^2 \frac{1}{x} dx$ with $n = 4$
- f) Find $y(0.1)$ using Taylor series method correct upto 4 decimal places for $y' = x - y^2$, $y(0) = 1$

Q.4

Attempt **any three** of the following:

15

- a) Given the following data find two regression lines :

| | Mean | Standard Deviation |
|---|-----------|--------------------|
| X | 40 | 10 |
| Y | 6 | 1.5 |
| | $r = 0.9$ | |

- b) Given the two regression lines as
- $4x - 5y = -33$
- and
- $20x - 9y = 107$
- , find the average of
- x
- and
- y
- and correlation coefficient between
- x
- and
- y
- .

- c) Solve the given LPP by graphical method :

Minimise $Z = 40x + 80y$

Subject to

$$36x + 6y \geq 108$$

$$3x + 12y \geq 36$$

$$20x + 20y \geq 100$$

$$x, y \geq 0$$

- d) Fit a parabola of the form
- $y = a + bx + cx^2$
- using least square method

| | | | |
|---|---|---|----|
| X | 0 | 1 | 2 |
| y | 1 | 6 | 17 |

- e) A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

- f) Find the Coefficient of correlation for the following data and comment on its value.

| | | | | | | | |
|---|----|---|---|----|----|----|---|
| X | 12 | 9 | 8 | 10 | 11 | 13 | 7 |
| Y | 14 | 8 | 6 | 9 | 11 | 12 | 3 |

Q.5

Attempt **any three** of the following:

15

- a) For a continuous random variable
- X
- , its probability density function is given by
- $f(x) = kx(2 - x)$
- ,
- $0 < x < 2$
-
- $= 0$
- , otherwise

Find k , $P(x < 1/2)$

- b) A random variable is sum of the numbers that appear when a pair of dice is rolled. Find probability distribution of the random variable. Also find mean and variance.
- c) Eight coins are tossed simultaneously. Find the probability of getting atleast six heads.
- d) Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored.
(i) More than 60 marks (ii) Less than 56 marks (iii) Between 45 and 65 marks
- e) The lifetime of a microprocessor is exponentially distributed with mean 3000 hours. Find the probability that
i) The microprocessor will fail within 300 hours
ii) The microprocessor will function for more than 6000 hours
- f) Define with an example:
 - i) Binomial Distribution
 - ii) Uniform Discrete distribution
 - iii) Normal distribution